Adrienn Dineva, Annamária R. Várkonyi-Kóczy, József K. Tar

WAVELET-BASED NOISE REMOVAL TECHNIQUE FOR REMOTELY-SENSED DATA

Filtering of the LiDAR data is challenging due to the complex distribution of the surface and the various types of contaminating noises. Also the collected data contain much information that requires the appropriate pre-processing in order to generate good DEMs. In this paper a new approach has been proposed for denoising and processing LiDAR data. The proposed method utilizes the advantages of multiresolution analysis and robust fitting. It has been shown that it excellently removes both additive noise and artifacts with retaining the important parts of the surface model. The method requires only low resolution levels and is able to avoid data loss.

Keywords: wavelet shrinkage, multiresolution analysis, remotely-sensed data, noise cancellation, robust fitting

INTRODUCTION

Noise cancellation is a primary issue of the theory and practice of signal processing. Several areas of engineering practice benefit from such algorithms, for instance monitoring and fault detection applications, data mining, feature identification in satellite images, etc. LIDAR (LIght Detection And Ranging) is a remote sensing technique based on laser technology. It measures the two way travel time of the emitted laser pulses to determine the distance between the sensor and the ground. With LiDAR (Light Detection and Ranging) or airborne laser scanning instruments mounted in aircrafts it has become possible to directly map the elevation of the surface beneath the aircraft's flight path. Combined with a Global Positioning System (GPS) and an Inertial Measurement Unit (IMU), LIDAR can generate a three-dimensional (3D) dense, geo-referenced point clouds for the reflective terrain surface.

The database collected by LIDAR is a point cloud in three dimensions which contain points returned from the terrain objects, including ground, buildings, bridges, vehicles, trees, and other non-ground features. For many applications it is important to detect, separate, or removed the artifacts in order to generate the digital elevation model [1]. While other applications require the precise reconstruction of the objects. Thus pre-processing is an important step which includes operations such as remove of systematic errors, filtering, feature detection and extraction, etc. [1]. A comprehensive overview of the major applications of airborne and terrestrial laser scanning can be found in e.g. [2].

Filtering of the LiDAR data is challenging due to the complex distribution of the surface and the various types of contaminating noises. Also the collected data contain much information that requires the appropriate pre-processing in order to generate good DEMs (Digital Elevation Model) or DSMs (Digital Surface Model) [3][4].

Traditional approaches, such as linear filtering, can smooth the corrupted signal, but with weak feature localization and incomplete noise suppression. Nonlinear filters have been proposed to overcome these limitations.
Among the classical signal processing methods, wavelet-based noise reduction has been successfully applied to filter data, because it provides information at a level of detail, which is not available with Fourier-based methods [5]. Several studies have been carried out regarding the utilization of wavelet theory, for e.g. see [6]. The discrete wavelet transform analyses the signal at different frequency scales with different resolutions by reducing the signal into approximate and detail information. For removing noise, wavelet shrinkage employs nonlinear soft thresholding functions in the wavelet domain. The popularity of this nonparametric method is due to the excellent localization and feature extracting behavior. However several threshold estimators exists, it is still a challenging task to select the appropriate shrinkage method that fits to the type of signal and contaminating noise and further, is robust against impulse type noises [7][8][9]. The other major issue in noise reduction is minimizing the effects of extreme values or elements that deviate from the observation pattern (outliers) [10][11]. LiDAR signals contain detailed information and may also contain outliers and random noise, as well as speckle noises that should be filtered out in order to obtain good quality final results.

Several aerial reconnaissance tasks require real-time immediate processing of the data. In order to obtain a good quality result in short time, fast and improved denoising methods are needed. This paper presents a new adaptive shrinkage approach applying adaptive robust fitting technique in the wavelet domain. Using the advantages of multiresolution analysis and robust fitting, efficient denoising can be obtained at low resolution levels providing simultaneously high density impulse noise removal. The proposed algorithm has been tested on LiDAR elevation data. Simulation results demonstrate the applicability of the proposed scheme and ensures good performance by correctly removing the noises and also the spikes and artifacts.

### WAVELET SHRINKAGE

#### 1 Basics on Wavelet Transform

In this section we give short outline on the background of wavelet transform. The wavelet transform (WT) maps a time function into a two-dimensional function of \( \phi \) (scale) and \( \tau \) (translation of the wavelet function along the time axis). Signal \( s(t) \in L^2 \) is assumed to be a square integrable meaning

\[
\int |s(t)|^2 dt < \infty.
\]  

(1)

The continuous wavelet transform (CWT) of \( s(t) \) is given by [5]

\[
CWT = \frac{1}{\sqrt{\phi}} \int s(t) \psi \left( \frac{t-\tau}{\phi} \right) dt.
\]  

(2)

where \( \psi \) denotes the basic (mother) wavelet function. In comparison with the STFT (short time Fourier Transform), in the WT the window width changes mean the dilation or compression a carrier frequency \( \omega_0 \) becomes \( \omega_0 / \phi \) for a window width change from \( T \) to \( \phi T \). When the \( \psi \) is a discretization of \( \psi \) the discrete WT can be written as follows,
where k,m integers.

2 Principles of Multiresolution Analysis

The idea of multiresolution analysis (MRA) lies in dividing a signal $s(n)$ into a set of (frequency) scales of $2^{-k}$. Associating with each scale (frequency band) is a subspace of $V^k$. Formally, let’s consider a sequence of increasing nested spaces $\{0\} \subset V_1 \subset V_2 \subset \ldots \subset H$, that form the basic resolution structure. The scaling function is $\phi \in H$, and its integer $\tau_n \phi$ translates (\(n \in \mathbb{Z}\)) form an orthonormal basis for one of the subspaces $V_k$ of $H$.

Let $\{V_k : V_k \in H\}$ be an increasing sequence of subspaces and $\phi \in V_0$. The multiresolution analysis of the pair $(\{V_k\}, \phi)$ of $H$ is defined as follows according to [8]

- there is a function $\phi \in V_0$, such that $\tau_n \phi$ is an orthonormal basis for $V_0$, and
- if any arbitrary function $f \in V_k$, then $D_2 f \in V_{k+1}$ (dilation invariance), further,
- $\bigcup V_k = H$ and $\bigcap V_j = \{0\}$ (completeness).

Basically, the MRA representation can be constructed by by the integer translation and power of two dilations (denoted by $D_2$) of the scaling function $\phi$. Under the assumption that $(\{V_k\}, \phi)$ is a multiresolution analysis of $H$, let $L_{\phi,k}$ denote the representation associated with the sequence of functions $D_2 \tau_n \phi$. For an arbitrary function $f \in H$, it’s $k^{th}$ resolution representation can be written as [5][8],

$$L_{\phi,k} f = \{(f; D_2 \tau_n \phi)\} = \{(f \ast D_2 \tau_n \phi)(2^{-k} n)\}$$

(4)

a discrete sequence of dilations and translation. Generating an orthonormal basis requires orthogonality between the levels, e.g. the subspaces $V_k$ are not orthogonal to each other. It is necessary to define an auxiliary sequence of subspaces $\{W_k: W_{k+1} \in H\}$, complementary to $V_k$ and orthogonal to $V_{k+1}$, for each level $k$ that characterizes the difference between $V_k$ and $V_{k+1}$ - which are the wavelet subspaces, so

$$V_{k+1} = V_k + W_k,$$

and

$$V_k - W_k.$$

As the scaling function $\phi$, the candidate analyzing wavelet is also a member of $V_i$, thus there are unique sequences (the scaling filter coefficients) $\{h_n\} = L_{\phi,0}$ and (the wavelet filter coefficients) $\{g_n\} = L_{\phi,1}$, such that, we can formulate the scaling function [8]

$$\phi(t) = \sum_n h_n \phi(2t - n)$$

(5)

and the wavelet function [8]

$$\psi(t) = \sum_n g_n \phi(2t - n)$$

(6)
3 Wavelet Shrinkage

The first step of wavelet shrinkage is the decomposition of signal into the wavelet (detail and approximate) coefficients, as described in the previous section. The general idea behind wavelet shrinkage is to replace these coefficients with small magnitude to zero (hard thresholding), or set their value to the threshold level [12]. Then, the reconstruction process performs the inverse discrete wavelet transform (IDWT). Most of the widely known shrinkage methods construct nonlinear threshold functions based on statistical considerations. An effective and smoothness-adaptive method (SureShrink) is proposed to thresholds each dyadic resolution level using the principle of Stein’s Unbiased Estimate of Risk [13][14]. The universal bound thresholding rule also provides good results with low computational complexity [13]; the rule is defined, as follows,

$$\eta = \sigma_{MAD} \sqrt{2 \log s_i},$$

(7)

where $$\sigma_{MAD} = \frac{\text{median}(\epsilon_i)}{0.6745}$$ denotes the absolute median deviation. The Heuristic Sure thresholding rule is introduced by a heuristic combination of the SureShrink and the universal bound [13];

$$\eta = \begin{cases} 
\eta_{UB} & \text{if } p \geq q \\
\min(\eta_{Sure}, \eta_{UB}) & \text{if } p < q 
\end{cases}$$

(8)

where $$p = \frac{m-k}{k}$$ and $$q = (\log_2 k)_0^2$$, $$m = \sum_{i=1}^{s} h_i^2$$. The Minimax estimator is also a preferred technique [9], the rule is given by

$$\eta = \begin{cases} 
\sigma_{MAD} (0.3936 + 0.1829 \log_2 s) & \text{if } s > 32 \\
0 & \text{if } s < 32 
\end{cases}$$

(9)

The specific choice of the wavelet function, decomposition level, and thresholding rule allows to construct many different shrinkage procedures. Further, the details of signal and also the type of contaminating noise.

ADAPTIVE SMOOTHING IN THE WAVELET TRANSFORM DOMAIN

1 Robust Fitting and Outlier Detection

Many works have been published on eliminating the outliers during the signal pre-processing. It has been proven, that the robust local polynomial regression technique is a very powerful technique for such problems [15]. At first, consider the classical noise suppression problem:

$$y_i = f(t) + \epsilon_i \quad i=1,...,s$$

(10)

where $$y_i$$ is the observed noisy data and $$\epsilon_i$$ represents the random noise, which is an independent and identically distributed (iid) process, and (t) stands for time. Let $$f$$ denote the unknown function. The principle of the local polynomial regression (loess) procedure can be summarized as follows. Function $$f(t)$$ can be approximated by fitting a regression surface to the data by determining a local neighbourhood of an arbitrary $$(t_0)$$. These neighbouring points are weighted depending on their distance from $$(t_0)$$. The closer points get larger weights [16]. The estimate $$\hat{f}$$ is obtained by fitting a linear or quadratic polynomial using the weighted values from the neighbourhood. Because this procedure relies on least squares regression, it is known that this
is vulnerable to outliers that can significantly degrade the result. In order to introduce robustness in the procedure an iterative reweighting is proposed with bisquare method [16]. Detailed description of these procedures can be read in [17].

2 Noise-cancellation of Remotely-sensed Data

In order to produce a good elevation model from LiDar data that meets the required accuracy an appropriate noise cancellation method should be applied. There are several sources of noise that can occur in airborne laser scanning systems that can distort the data. In previous work of the authors, it has been shown that the application of robust fitting in the wavelet transform domain can remove excellently different types of contaminating noises [18]. In the present paper we demonstrate that the improved version of the previous method is suitable for preprocessing LiDar data. The proposed method includes the following steps. Firstly, the raw data of the vertical direction is decomposed with orthogonal wavelet functions that correctly divide the detail and approximate coefficients of the signal. Then, a local polynomial regression curve is fitted on the coefficients with $w_i$ weights. After computing the residuals the robust weights are calculated with bisquare function and the fitting is repeated with these new weights. Finally, the inverse discrete wavelet transform reconstructs the data from such modified coefficients.

3 Simulation Results

The performance of the proposed method has been tested on a set of raw LiDar point cloud obtained from [19][20] that corrupted with additive white Gaussian noise and impulse noises (Fig.1.). The dataset consists of 4772 points. The results have been compared with two other shrinkage algorithms. The simulation has been built by using Matlab8. The performance has been measured by the root mean square error (RMSE) and the signal to noise ratio (SNR), given by the formula below

$$\text{SNR}_{\text{dB}} = 10 \times \log_{10} \frac{\sigma^2_s}{\sigma^2_n}$$

(11)

where $\sigma^2_s$ denotes the variation of the signal after denoising and $\sigma^2_n$ is the variation of the eliminated noise. For the decomposition an orthogonal symlet has been applied [5]. The results are summarized in Table 1. Fig. 2. shows the result of denoising with the Heuristic Sure method using 5 levels of decomposition while Fig. 3. displays the result for using minimax shrinkage rule at 8 level of decomposition. It can be observed that the classical shrinkage methods cannot handle the outliers. In Fig 4. the performance of the proposed method can be observed using 3 levels of decomposition.
Figure 1.: Raw LiDar data with additive white Gaussian noise and impulse – type noises

Figure 2.: Result of noise removal with Heuristic Sure method.
Figure 3.: Result of noise removal with Minimax shrinkage method.

Figure 4.: Result of noise removal with robust fitting in the wavelet transform domain.

<table>
<thead>
<tr>
<th></th>
<th>Original noisy data</th>
<th>Denoised data (heursure)</th>
<th>Denoised data (minmax)</th>
<th>Denoised data (robust fitting in wavelet domain)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR [dB]</td>
<td>3.29891E+01</td>
<td>3.7991270E+01</td>
<td>4.00261E+01</td>
<td>5.75986E+01</td>
</tr>
<tr>
<td>RMSE</td>
<td>1.54890E+00</td>
<td>1.0324</td>
<td>1.1230</td>
<td>0.532</td>
</tr>
<tr>
<td>Time [s]</td>
<td>1.11579E+00</td>
<td>1.79541E+00</td>
<td>1.02549E+00</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Simulation Results
CONCLUSIONS

In this paper a new approach has been proposed for denoising and processing LiDar data. The quality of the point cloud data produced by airborne lase scanning depends on several factors for instance the GPS and Inertial Measurement Unit accuracy, angular accuracy, extended GPS base lines and boresight calibration, etc. LiDAR signals contain detailed information and may also contain outliers and random noise, as well as speckle noises that should be filtered out in order to obtain good quality final results.

The proposed method utilize the advantages of multiresolution analysis and robust fitting. It has been shown that it excellently removes both additive noise and artifacts with retaining the important parts of the surface model. The method requires only low resolution levels and is able to avoid data loss.

ACKNOWLEDGEMENT

This work has been sponsored by the Hungarian National Scientific Research Fund (OTKA 105846).

REFERENCES

A légi lézerszkennelésből (airborne LiDAR – Light Detection and Ranging) nyert adatok zajszűrésére a kívánt pontosságú digitális terepmodellelőállításához napjainkban is kihívást jelent. A LiDar pontfelhő számos forrásból eredő torzításokat tartalmazhat. Jelen munkában a LiDar adatok előfeldogozásához alkalmas wavelet-alapú eljárást mutatunk be. A wavelet segítségével alacsony felbontási szinten is jól elkülöníthetők a zaj és a jel fontos részleteit tartalmazó atomok. Ezekre alkalmazva a robusztus illesztést az extreme értékek, torzítások is eltávolíthatók szemben a klasszikus shrinkage eljárásokkal.

Kulcsszavak: wavelet transzformáció, multirezolúciós felbontás, távérzékelt datok, zajszűrés, robusztus illesztés
